

A comparison of the fatigue calculation process according to Swedish and American textbooks for higher education

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ABSTRACT: Fatigue failure results from repeated localised plastic deformations in a machine member caused by variable loads and typically occurs at stress levels significantly lower than the yield strength of the material, usually after thousands – even millions – of cycles of minute yielding that often exists only on a microscopic level. Fatigue fractures start with a tiny (usually microscopic) crack at a critical area of high local stress, which enlarges until the section is sufficiently weakened so a final fracture occurs on one final load. Highly localised plastic yielding can be the start of a fatigue failure, notably in holes, sharp corners, threads, keyways, surface scratches and corrosion. Although the phenomena of fatigue has been known for over 150 years, such failures are very common; over 80% of mechanical breakdowns are from fatigue. They do not give any visible warning, it is sudden, total and catastrophic. As such, knowledge of fatigue calculations is very important for every machine designer. The author has found different methods and equations for fatigue calculations in Swedish and American higher education textbooks, giving different results. In the article, the author focuses on different shaft designs for infinite fatigue life that of different cyclic loading, using Swedish and American methods to compare the results.

INTRODUCTION

Often, machine members are found to have failed under action of variable (or repeated) stresses caused by variable loads. Between 80% and 90% of mechanical (breakdowns) failures are caused by fatigue [1]. The variable stresses can be one of the following:

- Fully reversed;
- Repeated;
- Fluctuated (see Figure 1) [2].

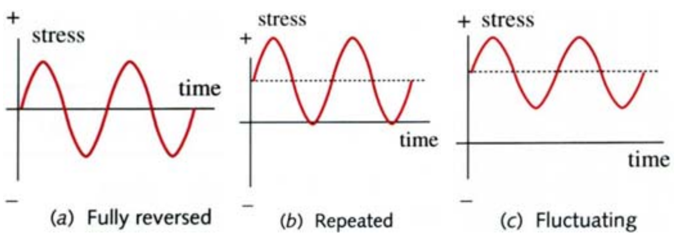

Figure 1: Different types of variable stresses [2].

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Analyses have shown that the actual maximum stresses are many times below the yield strength of the material. The most distinguishing characteristic of these failures is that the stresses have been repeated a very large number of times, thousand or millions of cycles. Hence, the failure is called a *fatigue failure* [3].

A fatigue failure, or *progressive fracture*, which is the more appropriate term to use, begins with a small crack. The initial crack is so minute that it cannot be detected by the naked eye. The crack develops at a point of discontinuity in the material, such as a change in cross section, a keyway, sharp corners, threads, holes, surface scratches and corrosion (see Figure 2). Once a crack has been initiated, the stress-concentration effect becomes greater and the crack progresses more rapidly. As the stressed area decreases in size, the stress increases in magnitude until the remaining area fails suddenly [3].

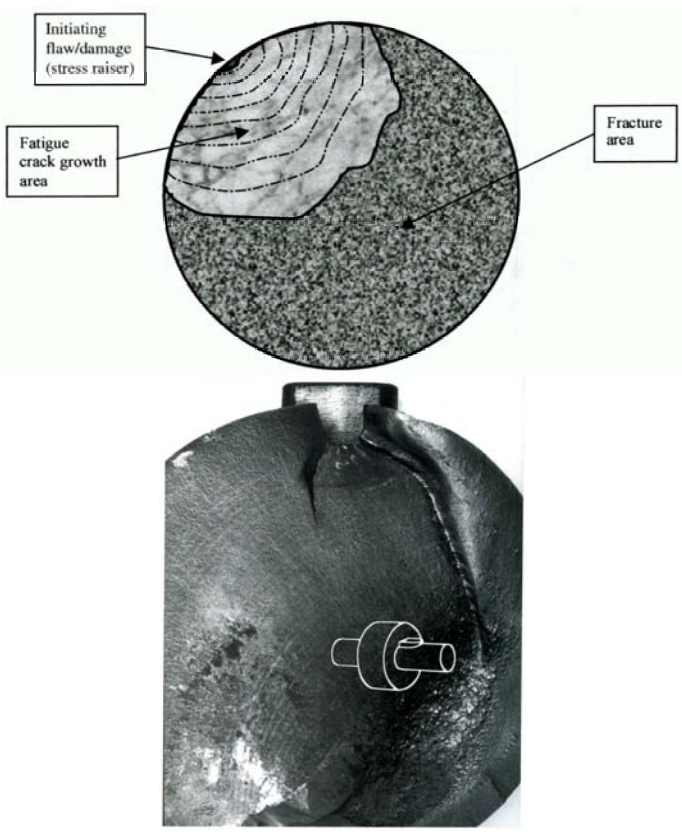


Figure 2a (top): Typical fatigue failure (schematic) [4]. Figure 2b (bottom): Fatigue fracture of a shaft with crack developed at a keyway [5].

When machine parts fail statically, they usually develop a very large deflection because the stress has exceeded the yield strength. Therefore, many static failures give visible warnings in advance. However, fatigue failure gives no warning; it is sudden and total, and hence dangerous (see Figure 3). It is relatively simple to design against a static failure because knowledge concerning this is comprehensive. Fatigue is a much more complicated phenomenon. Current engineering practice relies heavily on the wealth of empirical data that have accumulated from fatigue tests of numerous materials in various forms and subjected to various kinds of loading [6].

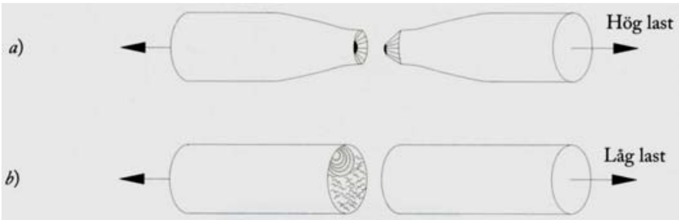


Figure 3: Shows a failure in a bar: (a) high load (static) and (b) low load (dynamic) [5].

The most known standardised fatigue test machine is a so-called R.R. Moore fatigue test machine. The machine is used to determine the fatigue strength characteristics of material under a standardised and highly restricted set of conditions. Figure 4 represents a standard R.R. Moore rotating-beam fatigue-testing machine [6]. In this machine, the specimen is subjected to pure bending by means of weights.

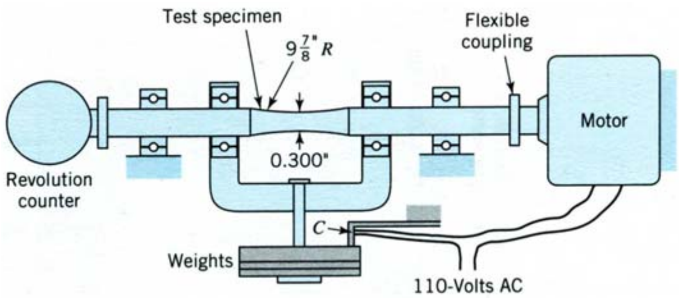


Figure 4: Moore's rotating-beam fatigue-testing machine [6].

The specimen in Figure 5 is very carefully machined and polished, with a final polishing in an axial direction so as to avoid circumferential scratches [3]. The highest level of stress is at the centre, where the diameter for the American standard is 0.300 inch (7.62 mm). The rest of the geometry is shown in Figure 5. Equivalent test-specimen geometry according to the Swedish standard is shown in Figure 6 [7]. According to ref. [8], the specimen must be made of row material of 20 mm diameter and the diameter in the centre of the specimen *d* must be 10 mm. But according to ISO 1099, the diameter *d* should be between 0.6 and 12.5 mm. Other fatigue testing machines are available for applying fluctuating or reversed axial stresses, torsional stresses or combined stresses to the test specimen.

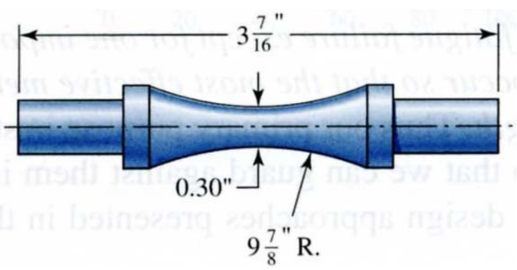


Figure 5: Test specimen geometry for the R.R. Moore rotating-beam machine [3].

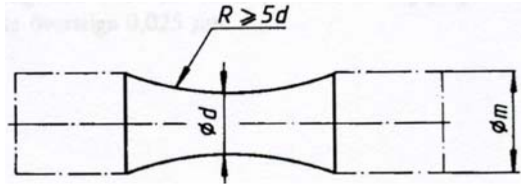


Figure 6: Test specimen geometry according to the Swedish standard [7].

A series of tests made with various weights and using test specimens carefully made to be as nearly identical as possible gives results that are plotted as S-N curves. Figure 7 shows an S-N curve for a type of steel, plotted on log-log coordinates to make the connection between the stress S and the lifecycle number easier to find. Ferrous materials have an endurance limit, defined as the highest level of alternating stress that can be withstood indefinitely without failure. The usual symbol for endurance limit is S_n. It is designated as S'_n, the prime indicates the special case of standard test illustrated in Figure 4, eg R.R. Moore rotating-beam fatigue-testing machine. As can be seen in Figure 7, there is a *knee* in the curve close to N = 10^6 cycles and the endurance limit is constant after the *knee*. This means that the ferrous materials have infinite life length above 10^6 cycles. There are S-N curves for other metals like aluminium and magnesium alloys. The characteristics of these metals are quite different from each other and from steel. For example, for various aluminium alloys, a sharply defined *knee* and, therefore, true endurance limit is absent. Fatigue strength at 10^8 or 5 x 10^8 cycles is often utilised.

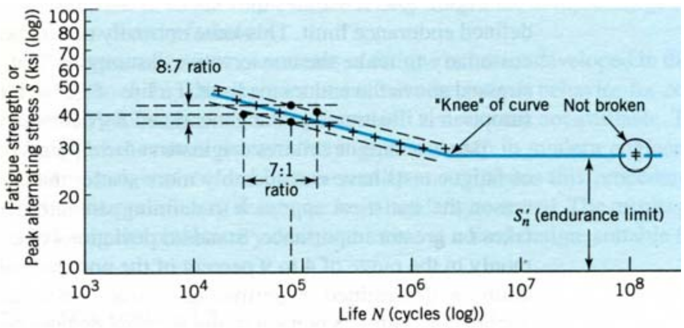


Figure 7: S-N plots (log-log coordinates) of representative data for 120 Bhn steel [6].

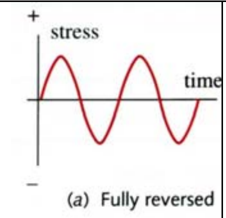
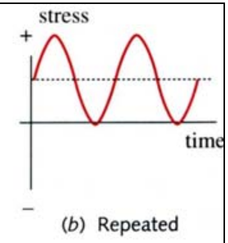
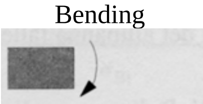
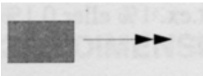
The endurance limit S'_n and S_n are used only in the American textbooks and it is the endurance limit for fully reversed loads. The stress from repeated and fluctuating loads can be calculated from S'_n and through σ_m-σ_a curves (mean stress-alternating stress curves). In the Swedish textbooks, there are tables listing the endurance limit depending on the type of loads (axial, bending or torsional) and depending if it is fully reversed or repeated (see Table 1).

The American and the Swedish methods of fatigue calculation are presented below. Table 2 presents the name and designation in the textbooks of both countries. It should be noted that the calculations presented in this article are from textbooks for the basic level courses of machine design and/or strength of materials.

THE FATIGUE CALCULATION PROCESS IN AMERICAN TEXTBOOKS

The calculation equations and process presented here are taken from Juvinall and Marshek [6]. This textbook does not take the temperature factor into account, nor do the Swedish textbooks; therefore, the calculation comparison is undertaken for materials at room temperature. A comparison is made for infinite life, eg for 10^6 number of cycles or more for steel. It is assumed that the materials are ductile. Table 3 presents some factors that will be used for other types of loading than the test (eg axial and torsion). There is also a factor for sizes bigger than the test specimen.

Table 1: Designation for different tests (stresses) in Swedish textbooks.

Load		
Axial 	$0 \pm \sigma_u$	$\sigma_{up} \pm \sigma_{up}$
Bending 	$0 \pm \sigma_{ub}$	$\sigma_{ubp} \pm \sigma_{ubp}$
Torsional 	$0 \pm \tau_{uv}$	$\tau_{uvp} \pm \tau_{uvp}$

Reversed axial or *push-pull* loading gives endurance limits that are about 10% lower than rotational bending. Furthermore, if the axial load is just a little off centre, slight bending will occur and the stress on one side will increase. In this case, the endurance limit will be 20-30% lower than the rotational bending, eg the gradient factor C_G = 0.9 for pure axial loading of precision parts and C_G = 0.7 to 0.9 for the axial loading of non-precision parts (see Table 3).

Table 3: Generalised fatigue strength factors for ductile materials [6].

Type of Load	Bending	Axial	Torsion
Load factor C_L	1.00	1.00	0.58
Gradient factor C_G			
Diameter < 10 mm	1.0	0.7 to 0.9	1.0
10 mm < diameter < 50 mm*	0.9	0.7 to 0.9	0.9

*For (50 mm) < diameter < (100 mm), reduce these factors by about 0.1. For (100 mm) < diameter < (150 mm), reduce these factors by about 0.2.

The endurance limit in reversed torsion is about 58% of the endurance limit in reversed bending, eg the load factor C_L = 0.58. Experimental values for the ultimate torsional shear strength should be utilised if they are available. If not, then the following equations should be used:

$$S_{us} = 0{,}8 S_u \text{ (for steel)} \qquad (1)$$
$$S_{us} = 0{,}7 S_u \text{ (for other ductile metals)} \qquad (2)$$

All the discussions concerning fatigue strength have assumed the surface to have a special *mirror polish* finish like in the test specimen. However, the surface finish is dependent on the manufacturing process and if there will be damage to the surface. Figure 8 gives the estimated values of the surface factor C_S for steel parts.

The three factors presented above are involved in the estimation of endurance limits, with equation (3) being used in the case of bending, axial or torsional load, which is as follows:

$$S_n = S_n' \, C_L \, C_G \, C_S \qquad (3)$$

using $S_n' = 0.5 S_u$ in the case of a lack of better data.

The load factor, C_L, and the gradient factor, C_G, can be found in Table 3, while the surface factor, C_s, can be estimated from the diagram presented in Figure 8.

Table 2: Fatigue names and designations in American and Swedish textbooks.

Name	American Designation	Swedish Designation
Standard fatigue strength for rotating bending	S_n'	---
Endurance limit for axial, bending and torsional loads (fully reversed, repeated and fluctuating)	S_n	σ_u or τ_u, but there are different designations depending on the type of the load and the variable stress (see Table 1)
Load factor	C_L	---
Gradient factor (size factor)	C_G	δ
Surface factor	C_s	κ
Size factor due to technology	---	λ
Notch sensitivity factor	q	q
Stress concentration factor	K_t	K_t
Fatigue stress concentration factor	K_f	K_f
Alternating stress	σ_a	σ_a
Mean stress	σ_m	σ_m
Ultimate strength	S_u	R_m (or σ_B*)
Yield strength	S_y	R_{eL} or $R_{p0.2}$ (or σ_s*) * old designations
Safety factor	SF	S_a if σ_m = constant S_m if σ_a = constant S_{am} if σ_a/σ_m = constant

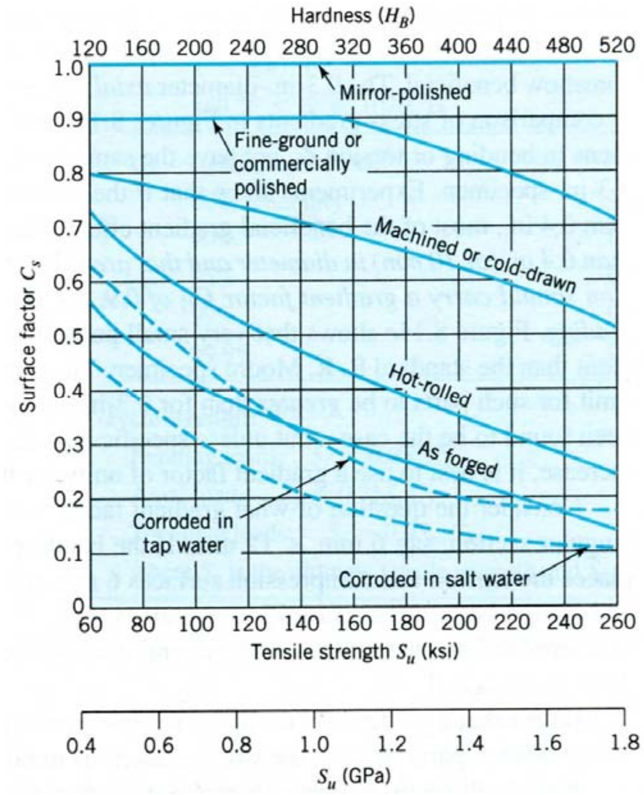


Figure 8: Surface factors for steel parts [6].

The static concentration factor K_t can be estimated from special diagrams depending on the part's (shaft's) geometry and the type of load (bending, axial or torsional).

Figure 9 shows an example of a diagram for estimating K_t for a shoulder shaft loaded by a bending moment. The diagrams are the same in the Swedish and American textbooks (see refs [1] and [6]).

The fatigue stress concentration factor, $K_{f,}$ is calculated from the following equation:

$$K_f = 1 + (K_t - 1)q \qquad (4)$$

where q is the notch sensitivity factor and is estimated from Figure 10.

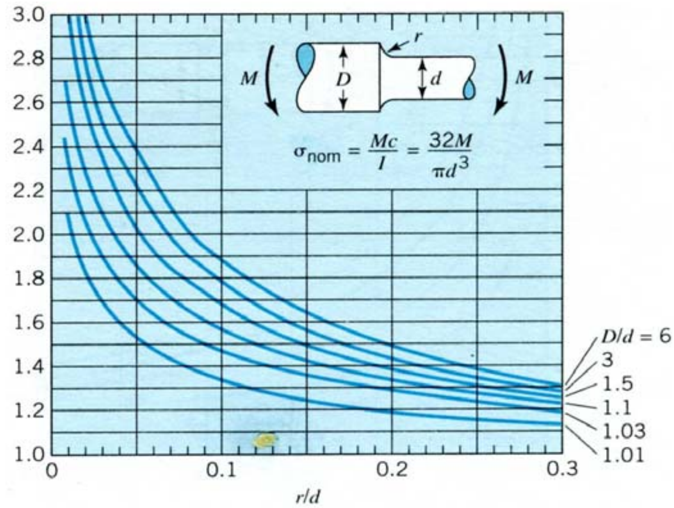


Figure 9: Diagram for estimating K_t for a bended shoulder shaft [6].

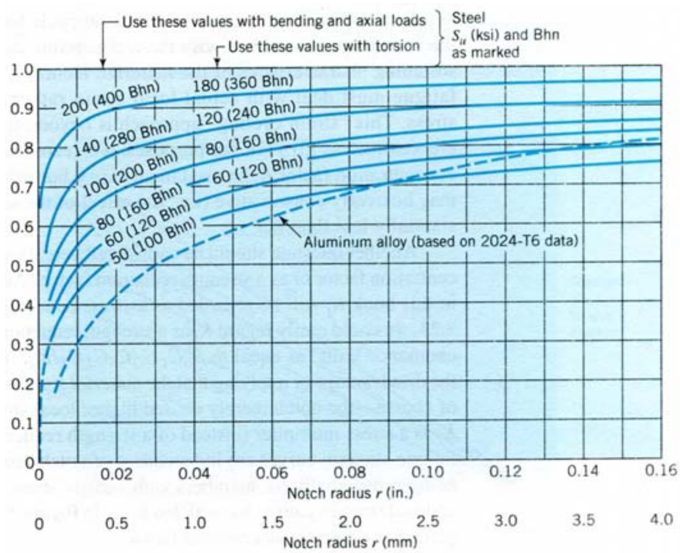


Figure 10: Diagram for estimating q in the American textbooks [6].

As stated before, the stresses can be completely reversed (with zero as the mean stress) or a combination of static plus completely reversed. Figure 11 presents the mean stress, alternating stress, maximum stress and minimum stress.

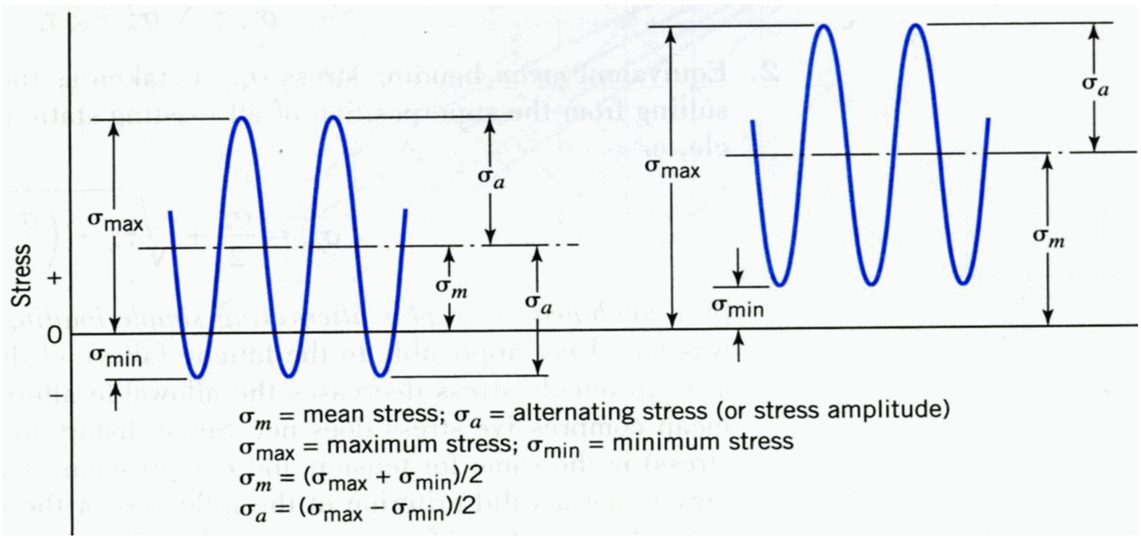


Figure 11: Different types of stress.

The materials alternating normal or shear stress/strength (σ_a or τ_a), and/or the mean normal or shear stress/strength (σ_m or τ_m) can be calculated from diagrams like those shown in Figure 12. The alternating and the mean stresses in a part (a shaft) depends upon the material's endurance limit, ultimate limit and yield limit and, of course, on the type of load and the relationship σ_a/σ_m or τ_a/τ_m.

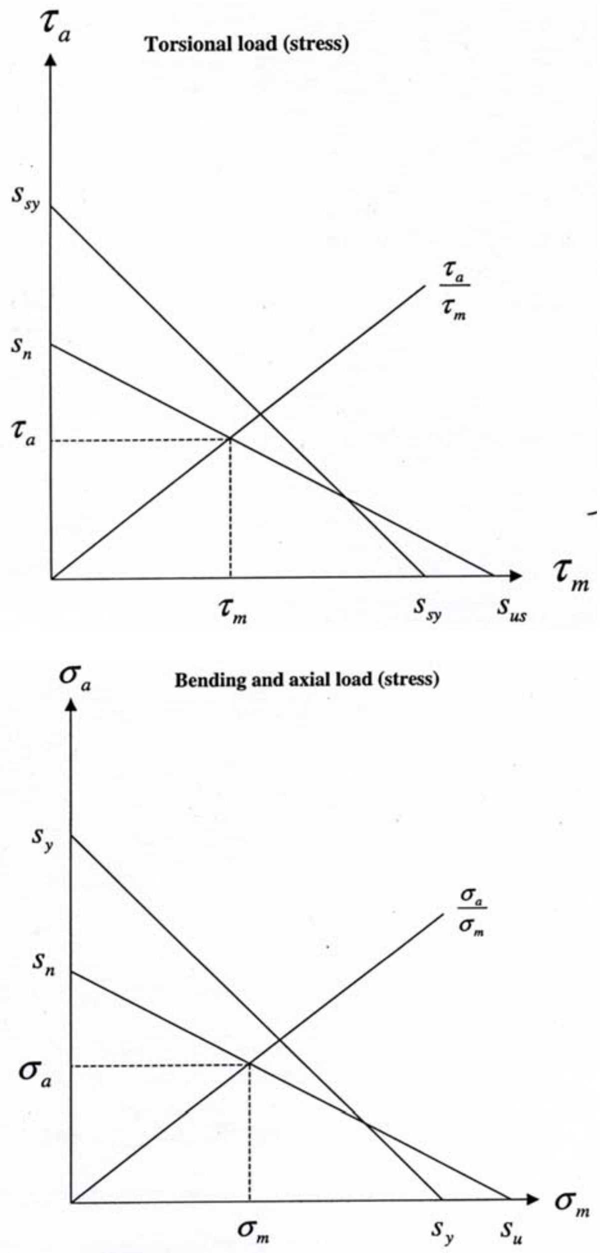


Figure 12: Diagrams for calculation (estimation) of σ_a and τ_a or σ_m and τ_m.

The alternating stress in a shoulder shaft, for example, loaded by a variable torsional load, can be calculated from the following equation:

$$\frac{\tau_a}{SF \cdot K_f} = \frac{16T_a}{\pi d^3} \tag{5}$$

where: T_a = Alternating torque; τ_a = Alternating shear stress; SF = Safety factor; K_f = Fatigue stress concentration factor.

An equivalent equation for bending stress in a similar shaft loaded by alternating bending moment can be calculated by:

$$\frac{\sigma_a}{SF \cdot K_f} = \frac{32M_a}{\pi d^3} \tag{6}$$

where: M_a = alternating bending moment; σ_a = alternating stress.

The axial stress in a shaft like above is as follows:

$$\frac{\sigma_a}{SF \cdot K_f} = \frac{4P_a}{\pi d^2} \tag{7}$$

where: P_a is alternating axial force.

THE FATIGUE CALCULATION PROCESS IN SWEDISH TEXTBOOKS

Unlike the American process of calculation of the ultimate limit from the material's rotational bending stress and the factors presented above, the Swedish process requires data tables showing the material's ultimate limit in different types of loading. Table 4 lists data for some standard steel types used in Sweden [1]. Unfortunately, the material codes are according to an old Swedish system and not the new Euro code.

In the case of a lack of data, for steel, the following equations should be used:

$$\sigma_{up} \approx 0.85\,\sigma_u \tag{8}$$
$$\sigma_u \approx 0.8\,\sigma_{ub} \tag{9}$$
$$\tau_{uv} \approx 0.58\,\sigma_{ub} \tag{10}$$
$$\tau_{uvp} \approx \tau_{uv} \tag{11}$$

The data from the data table is placed in a so called Haigh-diagram as shown in Figure 13. Note that in this figure there is a *knee* at the endurance limit at σ_{up} to compare with the endurance limit in Figure 12. This difference will make some differences in the calculated result depending on relationship of σ_a/σ_m.

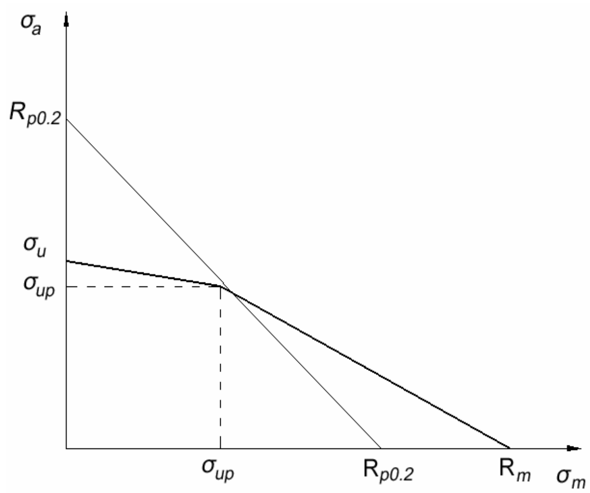


Figure 13: Haigh-diagram.

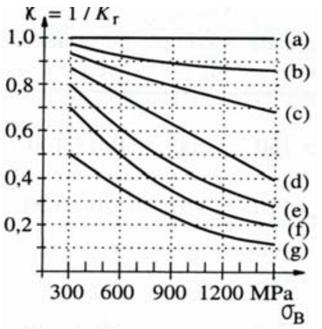
There are three reduction factors which must be indicated in the Haigh-diagram. They are as follows:

- Surface factor κ: Figure 14 gives the estimated values of surface factor for surface finished applied on steel (compare with Figure 8);

Table 4: Data for ultimate limits for some (Swedish) standard steels [1].

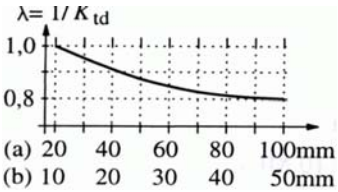
Material Code Steel	Axial Load		Bending Load		Torsional Load			
	Fully reversed $0\pm\sigma_u$ [MPa]	Repeated $\sigma_{up}\pm\sigma_{up}$ [MPa]	Fully reversed $0\pm\sigma_{ub}$ [MPa]	Repeated $\sigma_{ubp}\pm\sigma_{ubp}$ [MPa]	Fully reversed $0\pm\tau_{uv}$ [MPa]	Repeated $\tau_{uvp}\pm\tau_{uvp}$ [MPa]	Ultimate Strength [MPa]	Yield strength [MPa]
141312-00 Untreated	±110	110±110	±170	150±150	±100	100±100	360	240
141450-1 Normalised	±140	130±130	±190	170±170	±120	120±120	430	250
141510-00 Untreated	±230	---	---	---	---	---	510	320
141550-01 Normalised	±180	160±160	±240	210±210	±140	140±140	490	270
141650-01 Normalised	±200	180±180	±270	240±240	±150	150±150	590	310
141650 Tempered	---	---	±460	---	---	---	860	550

- Size factor due to technology λ: Figure 15 gives the estimated values of factor λ due to the size (diameter or thickness) of the raw material;
- Gradient factor (size factor) δ: Figure 16 gives the estimated values of factor δ due to the size /diameter or thickness) of the part (this can be compared with gradient factor C_G in Table 3).



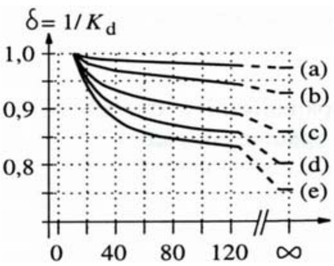
(a) Polished surface
(b) Grinded
(c) Rough machined
(d) Fillet (groove) according standard test
(e) Rolling skin
(f) Corrosion in potable water
(g) Corrosion in salt water

Figure 14: Surface factor κ [1].



(a) diameter in case of round cross section
(b) Thickness in case of rectangular cross section

Figure 15: Size factor due to technology λ [1].



Steel with ultimate strength R_m =
(a) 1500 MPa
(b) 1000 MPa
(c) 600 MPa
(d) 400 MPa
(e) Aluminum alloys

Factor δ = 1 if $K_f > 1$ is used.

Figure 16: Gradient factor (size factor) δ [1].

When all the three reduction factors are estimated, they are multiplied by σ_u and σ_{up} (alternative τ_{uv} and τ_{uvp}), as shown in Figure 17, eg $\lambda\delta\kappa\sigma_u$ and $\lambda\delta\kappa\sigma_{up}$ (alternative $\lambda\delta\kappa\tau_{uv}$ and $\lambda\delta\kappa\tau_{uvp}$).

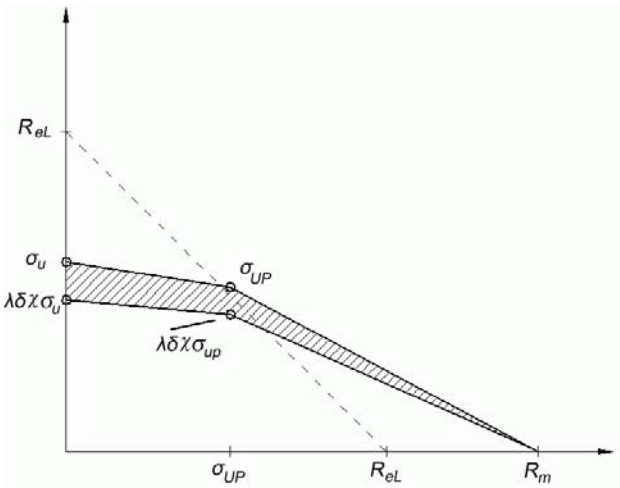


Figure 17: The three reduction factors are drawn in the Haigh-diagram.

The fatigue safety factor can be calculated using three different methods – depending on if σ_m is constant, σ_a is constant or σ_a/σ_m is constant. The three equations are presented below together with Figure 18:

$$S_a = \frac{AA'}{AP} \quad \text{if} \quad \sigma_m = \text{constant} \tag{12}$$

$$S_m = \frac{OB'}{OA} \quad \text{if} \quad \sigma_a = \text{constant} \tag{13}$$

$$S_{am} = \frac{OC'}{OP} \quad \text{if} \quad \sigma_a/\sigma_m = \text{constant} \tag{14}$$

The notch sensitivity factor q for the Swedish calculation process is assumed from Figure 19.

The fatigue stress concentration factor K_f is calculated by using equation 4. The torsional, bending and axial stresses are calculated in the same manner as in the American textbooks, eg by using equations (5), (6) and (7).

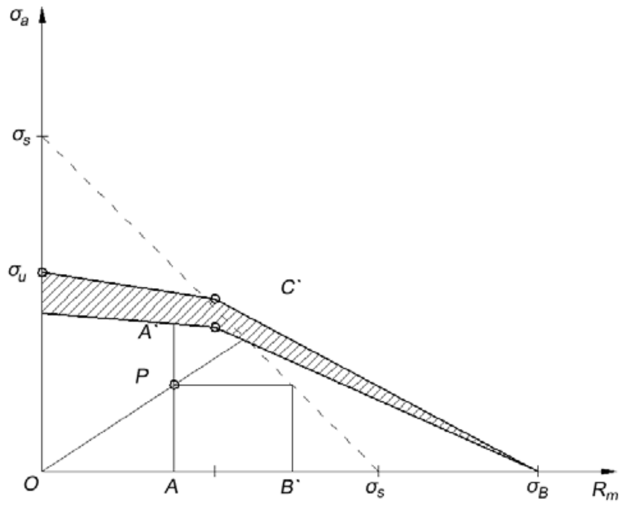
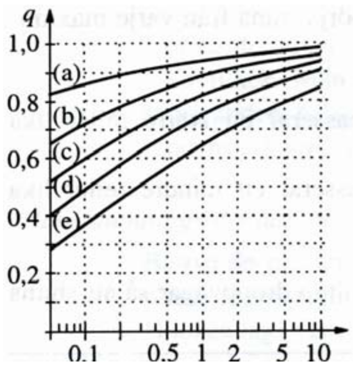


Figure 18: Calculation of the safety factor.



Fillet radius r in mm

Notch sensitivity factor q for steel with an ultimate strength of R_m =

(a) 1600 MPa
(b) 1300 MPa
(c) 1000 MPa
(d) 700 MPa
(e) 400 MPa

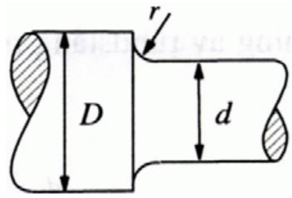
Figure 19: Notch sensitivity factor q in the Swedish text-books [9].

The equations and calculation processes, as presented above, are now applied to a shaft so as to find out if there is any differences in the results calculated by the American and the Swedish calculation processes. In order to have a better picture of the situation, different materials (steel) are utilised, as well as different types of loads (at each situation). A simple example of a step (shoulder) shaft is presented below.

Example: a Shoulder Shaft with Fixed Dimensions

The following example of a shoulder shaft with fixed dimensions (as shown in Figure 20) is subjected to the three loading types, eg bending, torsional and axial (one load type at a time). The load types are fully reversed, repeated and fluctuated.

The data is as follows:



D = 40 mm
d = 30 mm
r = 1.5 mm
Machined surface
Made of 50 mm-diameter row material
Safety factor SF = 2

Figure 20: The shoulder shaft.

Three different materials from the Swedish standard are used (see Table 4). Table 5 lists the three different materials.

Table 5: Some material (steel) according to the Swedish standard with the European equivalent [9].

Material Number	Swedish Material (Designation)	Replaced by European Standard (Codes)
1	141312-00	SS-EN 10 025 – s235JRG2 (1.0038)
2	141450-1	No replacement (out of standard)
3	141550-01	SS-EN 10 025 – E295 (1.0050)

The results from the different calculations (3 x 3 x 3 x 2 = 54 calculations) are presented in Table 6. As shown in the table, there are differences in the most of the cases of the results depending on the calculation processes. The Swedish calculation process yields better result (higher load) in most of the situations of the bending and torsion loads, while the American calculation process gives a better result in most of the situations regarding axial loads.

The Swedish calculation process gives a 5-25% stronger shaft in the case of bending loads. While in the case of axial loads, the American process seems to give a 0-50% stronger shaft, except for the material number 3 and in the case of repeated and fluctuated loads. In the case of torsional loads, the Swedish process gives 5-25% stronger shafts, except in the case of the repeated load for material number 2 and the fluctuated load for material number 3.

CONCLUSION

It is not always easy to change students' educational textbooks from one language to another, especially in the field of engineering. It has occurred many times that the calculation equations and processes can be different from one country to another. Many of the equations and calculation processes are built on the respective country's national standards. The national standard can be different in some fields from one country to another.

Table 6: The calculation results, American versus Swedish.

	Load type	Bending [Nm]			Axial [N]			Torsion [Nm]		
Material number	Calculation process	Fully reversed	Repeated	Fluctuated	Fully reversed	Repeated	Fluctuated	Fully reversed	Repeated	Fluctuated
1	American	107	79	61.7	25.6	18.9	14.8	140	112	84.6
	Swedish	112.5	100	66.7	17.2	17.2	14.8	155	141.2	92.9
2	American	123	90.4	66.6	29.1	21.4	15.8	165.7	147.3	88.4
	Swedish	125	103.4	70	22.4	20.8	16.4	185.8	145.1	96.8
3	American	138	102.5	71	32.9	24.3	16.8	186.5	148	109.7
	Swedish	158.4	113	75	28.4	25.2	20	216.7	158.7	104.5

In this article, the author shows the American and the Swedish versions of the equations and calculation processes for fatigue. A concrete example is shown that illustrates the differences in the results from these two methods for calculations in most of the cases concerning fatigue. It is not easy to say which of the calculation processes is closer to the reality, because the calculation equations are simple models of a complex reality.

In addition to that, it is assumed in the theory that the materials are perfect, the working environment is acceptable and the loads' magnitude and types are known. However, it is not always that situation in reality. In the calculations above, no factor for temperature is used in any of the two processes. It is known that even ductile materials behave as brittle materials in the case of low temperatures. Brittle materials are not suitable to use in machine parts that are subjected to fatigue stress.

The Swedish calculation process requires more material data besides the ultimate and yield strengths. It requires data on fully reversed, repeated and fluctuating limits for bending, axial and torsion loads. In this case, the result of this calculation process may be closer to the reality. The problem illustrates the lack of such data on many materials; only a few materials textbooks provide these data.

Furthermore, given that the difference in results in some situations were not that big, the American calculation process is preferred. The author's experience is that students find it easier to understand the American calculation process over the Swedish method. However, teaching both processes in the same course has shown that students become confused.

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